Gravity as Aliasing Residue: A Dual-Field Phase Model with Finite Sampling

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Abstract

We propose a phenomenological model in which the effective gravitational field arises as an aliasing residue of two underlying, phase-offset fields when they are represented at finite informational resolution. Concretely, we consider a dual-field description in which a "dielectric"-like potential Φ_D and an "electromagnetic"-like potential Φ_E combine into a single complex field

$$\Psi(x,t) = \Phi_D(x,t) + i\Phi_E(x,t) = A(x,t)e^{i\phi(x,t)}.$$

At infinite resolution, the local phase difference between the subfields is exactly resolved and does not generate an additional force. When the field is sampled through a finite-resolution map S_Δ—which may be interpreted as a coarse-grained spatial/temporal sampling or a finite information bandwidth—high-frequency structure in the relative phase becomes under-resolved and folds back into the resolved band as an aliasing contribution. We define an aliasing operator $\mathcal{A}_\Delta$ that explicitly captures this back-folded component and show that the associated "aliasing energy density"

$$\rho_{\text{alias}}(x,t) \propto |\mathcal{A}_\Delta[\Psi](x,t)|^2$$

naturally plays the role of a gravitational potential source. In a simple monochromatic toy model with a small phase offset $\Delta\phi$ between Φ_D and Φ_E, we obtain an effective gravitational strength

$$G_{\text{eff}} \propto 1 - \cos(\Delta\phi) \approx \frac{1}{2}(\Delta\phi)^2 \quad (\Delta\phi \ll 1),$$

identifying gravity with the quadratic residue of an otherwise perfectly balanced dual-field pair. This construction provides an explicit operator-level route from finite sampling and information loss to an emergent, attractive potential, suggesting a possible information-theoretic underpinning for gravity as seen in coarse-grained, low-energy physics.

1 Introduction

The standard description of gravity in modern physics is geometric: in general relativity, matter and energy curve spacetime, and test bodies follow geodesics in that curved geometry. Quantum field theory, by contrast, describes other interactions in terms of fields living on a background and their quanta. Bridging the conceptual gap between these pictures—and understanding whether gravity itself might admit a fundamentally informational or emergent description—remains an open problem.

In parallel, sampling theory and aliasing are completely understood in signal processing but are rarely treated as dynamical objects in fundamental physics. Given a continuous field $f(x)$ and a finite sampling scale Δ, the Nyquist-Shannon theorem tells us precisely when f can be perfectly reconstructed and when high-frequency structure "folds back" into the resolved band as aliasing. In engineering contexts, aliasing is usually an unwanted artifact. In this work we explore the opposite possibility: that an aliasing residue of deeper, more symmetric fields may be exactly what we macroscopically interpret as gravity.

We start from a minimal hypothesis:

- There exist two underlying, physically meaningful fields, which we denote Φ_D ("dielectric-like") and Φ_E ("electromagnetic-like"). Their detailed microphysical interpretation is left open.
- At a given spacetime point (x, t), these combine into a complex field

  $$\Psi(x,t) = \Phi_D(x,t) + i\Phi_E(x,t) = A(x,t)e^{i\phi(x,t)}, \tag{1}$$

  with amplitude A and overall phase ϕ.
- At infinite informational resolution, the system is exactly self-balanced: no standalone gravitational potential appears at this level.
- Physical observers, however, only have access to a finite-resolution representation of Ψ, modeled by a sampling/coarse-graining map S_Δ characterized by a scale Δ in time, space, or both.

The key move in this paper is to treat the action of S_Δ not as a passive loss of detail but as an active map that generates a well-defined aliasing term. Schematically, we decompose the full field into a "resolved" band and an "under-resolved" band,

$$\Psi = \Psi_{\text{res}} + \Psi_{\text{sub}}, \tag{2}$$

and define an aliasing operator $\mathcal{A}_\Delta$ that encodes how sub-resolution structure Ψ_{sub} folds into the resolved representation once sampling is imposed. The central proposal is then:

$$\Phi_g(x,t) \propto |\mathcal{A}_\Delta[\Psi](x,t)|^2, \tag{3}$$

where Φ_g plays the role of an effective gravitational potential. The corresponding acceleration field is obtained via

$$\mathbf{g}(x,t) = -\nabla\Phi_g(x,t). \tag{4}$$

This framing is intentionally agnostic about the microscopic origin of Φ_D and Φ_E. What matters for our purposes is the following structural statement:

If there exists a dual-field configuration whose internal phase structure is exactly self-cancelling at infinite resolution, and if physical observers necessarily interact with a finite-resolution representation of that configuration, then the mismatch between the perfect cancellation and the sampled representation generically generates a residual, attractive potential proportional to an aliasing norm.

In the rest of the paper, we make this statement precise. In Sec. 2, we formalize the dual-field complex representation and the notion of a local phase offset between subfields. In Sec. 3, we define a simple aliasing operator $\mathcal{A}_\Delta$ built from projection operators and sampling maps. In Sec. 4, we analyze a monochromatic toy model in which the phase offset enters as a parameter $\Delta\phi$ and show that the effective gravitational strength scales as

$$G_{\text{eff}} \propto 1 - \cos(\Delta\phi) \approx \frac{1}{2}(\Delta\phi)^2$$

for small $\Delta\phi$. In Sec. 6, we discuss how this construction might connect to existing emergent gravity and information-theoretic approaches, and what types of empirical or phenomenological constraints would be needed for such a model to be taken seriously.

2 Dual-Field Structure and Phase

We begin by making Eq. (1) more explicit. Let $\Phi_D(x,t)$ and $\Phi_E(x,t)$ be two real scalar fields defined on spacetime. We assemble them into a complex field

$$\Psi(x,t) = \Phi_D(x,t) + i\Phi_E(x,t). \tag{5}$$

Writing Ψ in polar form,

$$\Psi(x,t) = A(x,t)e^{i\phi(x,t)}, \tag{6}$$

defines the amplitude

$$A(x,t) = \sqrt{\Phi_D(x,t)^2 + \Phi_E(x,t)^2} \tag{7}$$

and overall phase

$$\phi(x,t) = \arctan\left(\frac{\Phi_E(x,t)}{\Phi_D(x,t)}\right). \tag{8}$$

For our purposes, the essential ingredient is not the absolute phase ϕ, but the phase structure between contributions that, at infinite resolution, are arranged to cancel in the sense of producing no net long-range force. A simple intuition-building case is to consider two monochromatic modes with a relative phase offset; this will be our starting point in Sec. 4. Before that, we introduce the finite-resolution map and the associated aliasing operator.

3 Finite-Resolution Sampling and the Aliasing Operator

Physical observers never access a field $\Psi(x,t)$ at infinite resolution. Instead, measurements, detectors, and coarse-grained descriptions effectively apply a sampling map

$$S_\Delta : \Psi \mapsto \Psi_\Delta, \tag{9}$$

where Δ denotes the characteristic resolution scale (spatial, temporal, or informational).

A convenient way to model sampling is through band-limited projectors. Let P_{res}^{Δ} be the projector onto modes with wavenumbers $|k| < \Lambda(\Delta)$, where $\Lambda(\Delta)$ is the Nyquist-like cutoff. Let $P_{\text{sub}}^{\Delta} = I - P_{\text{res}}^{\Delta}$ denote the complementary projector onto under-resolved modes. We then decompose

$$\Psi = \Psi_{\text{res}} + \Psi_{\text{sub}}, \tag{10}$$

with

$$\Psi_{\text{res}} = P_{\text{res}}^{\Delta}\Psi, \quad \Psi_{\text{sub}} = P_{\text{sub}}^{\Delta}\Psi. \tag{11}$$

Sampling modifies the representation of Ψ by discarding direct access to Ψ_{sub}. However, under-sampled components do not vanish—they fold back into the resolved band. This motivates the definition of an aliasing operator.

3.1 Definition of the Aliasing Operator

We define the aliasing operator

$$\mathcal{A}_{\Delta}[\Psi] \equiv P_{\text{res}}^{\Delta}\Psi_{\text{sub}} = P_{\text{res}}^{\Delta}P_{\text{sub}}^{\Delta}\Psi. \tag{12}$$

Several comments are important:

- $P_{\text{res}}^{\Delta}P_{\text{sub}}^{\Delta} \neq 0$ in general. Projecting a high-frequency mode onto a low-frequency basis produces nonzero leakage—the mathematical essence of aliasing.
- $\mathcal{A}_{\Delta}$ vanishes only in the infinite-resolution limit $\Delta \to 0$ (or $\Lambda \to \infty$).
- $\mathcal{A}_{\Delta}$ is linear and well-defined for any field admitting a Fourier representation.

The sampled field is then

$$\Psi_{\Delta} = \Psi_{\text{res}} + \mathcal{A}_{\Delta}[\Psi]. \tag{13}$$

Thus the observer never sees the true resolved band; they see

$$(\text{resolved structure}) + (\text{back-folded residue}).$$

3.2 Aliasing Energy Density

We now associate a scalar quantity to the aliasing component:

$$\rho_{\text{alias}}(x,t) \equiv \alpha|\mathcal{A}_{\Delta}[\Psi](x,t)|^2, \tag{14}$$

where α is a constant setting the units or coupling scale. This quantity is always:

- non-negative,
- resolution-dependent,
- zero only if no sub-resolution structure leaks into the resolved band.

3.3 Effective Gravitational Potential

We identify the effective gravitational potential as

$$\Phi_g(x,t) = C\rho_{\text{alias}}(x,t), \tag{15}$$

where C determines the strength of coupling to curvature or acceleration.

The corresponding gravitational acceleration is

$$\mathbf{g}(x,t) = -\nabla\Phi_g(x,t) = -C\nabla|\mathcal{A}_\Delta[\Psi](x,t)|^2. \tag{16}$$

This is the central operational statement of the model:

> *Gravity is the spatial gradient of the squared magnitude of the aliasing residue of two deeper fields when sampled at finite resolution.*

3.4 Interpretation

The model does not insert gravity by hand. Rather, gravity emerges because:

1. Two underlying fields cancel perfectly at infinite resolution.
2. Finite-resolution observers cannot resolve the phase structure that enforces this cancellation.
3. Loss of resolution produces an unavoidable back-folded residue.
4. The norm of that residue behaves like a potential generating attractive acceleration.

This mechanism is independent of the microphysical interpretation of the fields. It requires only:

dual-field structure + phase offset + finite sampling.

In Sec. 4, we evaluate this residue explicitly for a monochromatic dual-field configuration and obtain a closed-form expression for the effective gravitational strength.

4 Toy Monochromatic Dual-Field Model

To make the mechanism concrete, we consider a toy configuration where the underlying fields are monochromatic plane waves with a fixed phase offset. For simplicity, we work in one spatial dimension; the extension to higher dimensions is straightforward.

4.1 Dual-Field Configuration and Imperfect Cancellation

Let

$$\Phi_1(x,t) = Ae^{i(kx-\omega t)}, \quad \Phi_2(x,t) = Ae^{i(kx-\omega t+\Delta\phi)}, \tag{17}$$

with amplitude $A > 0$, wavenumber k, frequency ω, and constant phase offset $\Delta\phi$ between the two fields.

We now form a cancelling combination

$$\Psi_{\text{tot}}(x,t) = \Phi_1(x,t) - \Phi_2(x,t). \tag{18}$$

At the level of infinite precision, this represents two equal-amplitude contributions designed to annihilate each other when $\Delta\phi = 0$.

Using

$$1 - e^{i\Delta\phi} = 2ie^{i\Delta\phi/2}\sin\left(\frac{\Delta\phi}{2}\right), \tag{19}$$

we obtain

$$\Psi_{\text{tot}}(x,t) = Ae^{i(kx-\omega t)}\left(1 - e^{i\Delta\phi}\right) = 2A\sin\left(\frac{\Delta\phi}{2}\right)e^{i\theta(x,t)}, \tag{20}$$

where $\theta(x,t)$ is an unimportant overall phase. The magnitude of the cancelling combination is therefore

$$|\Psi_{\text{tot}}(x,t)| = 2A\left|\sin\left(\frac{\Delta\phi}{2}\right)\right|. \tag{21}$$

In the ideal, perfectly resolving theory, this residual can be driven arbitrarily small by enforcing $\Delta\phi \to 0$. In the aliasing picture, however, the relevant point is that this phase structure must be represented at finite resolution.

4.2 Action of the Aliasing Operator on a Single Mode

For a single monochromatic mode, the aliasing operator $\mathcal{A}_\Delta$ defined in Eq. (12) acts as an effective complex scalar on the mode:

$$\mathcal{A}_\Delta[\Psi_{\text{tot}}] = \eta(\Delta,k)\Psi_{\text{tot}}, \tag{22}$$

where $\eta(\Delta,k)$ encodes how strongly this particular (k,ω) configuration leaks into the resolved band at resolution Δ. We write

$$F(\Delta,k) \equiv |\eta(\Delta,k)| \in [0,1], \tag{23}$$

with $F \to 0$ when the mode is either fully resolved or fully filtered, and F maximal when it aliases most strongly into the resolved band.

Combining Eqs. (18) and (22), the aliasing contribution becomes

$$\mathcal{A}_\Delta[\Psi_{\text{tot}}(x,t)] = 2A\eta(\Delta,k)\sin\left(\frac{\Delta\phi}{2}\right)e^{i\theta(x,t)}. \tag{24}$$

Its magnitude is

$$|\mathcal{A}_\Delta[\Psi_{\text{tot}}(x,t)]| = 2AF(\Delta,k)\left|\sin\left(\frac{\Delta\phi}{2}\right)\right|. \tag{25}$$

4.3 Aliasing Energy and the $1 - \cos\Delta\phi$ Factor

Using the definition of the aliasing energy density, Eq. (14), we obtain

$$\rho_{\text{alias}}(x,t) = \alpha|\mathcal{A}_\Delta[\Psi_{\text{tot}}(x,t)]|^2 = \alpha \cdot 4A^2 F(\Delta,k)^2 \sin^2\left(\frac{\Delta\phi}{2}\right). \tag{26}$$

Using the trigonometric identity

$$\sin^2\left(\frac{\Delta\phi}{2}\right) = \frac{1}{2}(1-\cos\Delta\phi), \tag{27}$$

this becomes

$$\rho_{\text{alias}}(x,t) = 2\alpha A^2 F(\Delta,k)^2(1-\cos\Delta\phi). \tag{28}$$

Thus, for this monochromatic dual-field configuration, the aliasing energy is proportional to the familiar $1-\cos\Delta\phi$ residue: it vanishes for perfect phase alignment and grows with the phase mismatch between the cancelling fields.

4.4 Effective Gravitational Strength

Substituting Eq. (28) into the identification of the effective gravitational potential, Eq. (15), we obtain

$$\Phi_g(x,t) = C\rho_{\text{alias}}(x,t) = 2C\alpha A^2 F(\Delta,k)^2(1-\cos\Delta\phi). \tag{29}$$

In a coarse-grained description, this suggests an effective gravitational coupling of the form

$$G_{\text{eff}}(k,\Delta,\Delta\phi) = G_0 F(\Delta,k)^2(1-\cos\Delta\phi), \tag{30}$$

where G_0 collects numerical factors and units. Two structural features are immediate:

- $G_{\text{eff}} \to 0$ as $\Delta\phi \to 0$: perfectly phase-matched dual fields do not gravitate in this channel.
- G_{eff} is maximized when the phase mismatch and alias strength are both large; gravity is strongest when cancellation fails in a way that the finite-resolution observer cannot resolve.

4.5 Small Phase-Offset Limit

For small phase offsets, $\Delta\phi \ll 1$, we have

$$1-\cos\Delta\phi \simeq \frac{1}{2}(\Delta\phi)^2, \tag{31}$$

so that

$$G_{\text{eff}}(k,\Delta,\Delta\phi) \simeq \frac{1}{2}G_0 F(\Delta,k)^2(\Delta\phi)^2. \tag{32}$$

In this regime, the effective gravitational coupling is quadratic in the microscopic phase mismatch between the dual fields and weighted by the resolution-dependent aliasing factor $F(\Delta,k)^2$. The core statement of the toy model is then:

> *At fixed resolution and mode content, gravity measures the phase-mismatch residue of a dual-field configuration that would cancel exactly at infinite precision.*

5 Results and Interpretation

The toy model of Sec. 4, together with the operational definition of the aliasing operator $\mathcal{A}_\Delta$ introduced in Sec. 3, allows a compact formulation of the central result:

> *Gravity emerges as a finite-resolution measure of the failure of dual-field cancellation, quantified as the phase-mismatch residue that survives coarse-graining.*

Mathematically, the effective gravitational potential for a monochromatic dual-field configuration takes the form

$$\Phi_g(x,t) = 2C\alpha A^2 F(\Delta,k)^2(1-\cos\Delta\phi), \tag{33}$$

and the corresponding effective gravitational coupling is

$$G_{\text{eff}}(k,\Delta,\Delta\phi) = G_0 F(\Delta,k)^2(1-\cos\Delta\phi). \tag{34}$$

These expressions contain three structural elements:

5.1 (i) Phase Mismatch as Gravitational Seed

The combination $1-\cos\Delta\phi$ measures the departure from perfect cancellation of the dual fields. For $\Delta\phi = 0$, the two fields annihilate at all resolutions and no gravitational signal appears: $G_{\text{eff}} = 0$. As the mismatch grows, the emergent coupling increases smoothly and reaches a maximum at $\Delta\phi = \pi$.

Thus, phase differences that would be negligible at infinite precision become dynamically relevant when evaluated through a finite-resolution observer.

5.2 (ii) Aliasing Bandwidth as Geometric Amplifier

The factor $F(\Delta,k)$ encodes how strongly a given mode leaks into the resolved band. This factor turns out to control the magnitude of the gravitational coupling far more strongly than the raw phase mismatch alone. Modes that sit just beyond the resolution cutoff alias most efficiently, so they contribute most to G_{eff}.

This establishes a channel through which resolution-dependent geometry regulates gravitational strength.

5.3 (iii) Quadratic Form in the Small-Mismatch Limit

For $\Delta\phi \ll 1$, we obtain

$$G_{\text{eff}} \simeq \frac{1}{2} G_0 F(\Delta,k)^2(\Delta\phi)^2, \tag{35}$$

revealing a simple and universal structure: gravity measures the square of the microscopic cancellation error, modulated by finite-resolution aliasing. This quadratic residue is the defining signature of the aliasing model.

5.4 Interpretation

The essential physical picture is as follows. Two fields whose amplitudes and phases are tuned to cancel exactly at the microscopic level generate no gravitational response in an ideal, fully resolving theory. However, a finite-resolution observer cannot fully represent the microscopic phase pattern of the cancellation. As a result, the coarse-grained fields do not cancel perfectly, and the residual pattern re-enters the resolved band through aliasing. We identify the energy associated with this reintroduced residue with the gravitational channel.

In this view,

> *gravity quantifies the precision gap between the microscopic dual-field configuration and the macroscopic resolution scale.*

Unlike conventional quantum-gravity pictures, the present construction does not require quantizing the gravitational field or modifying general relativity directly. Instead, gravitational behavior emerges from the structure of finite-resolution sampling applied to otherwise cancellation-symmetric quantum fields.

The explicit appearance of the alias residue $(1 - \cos \Delta\phi)$ and the resolution bandpass $F(\Delta, k)$ provides a route for concrete predictions: different resolutions and mode spectra imply different strengths of the apparent gravitational interaction. This makes the framework falsifiable in principle and places it within the scope of precision tests of modified gravitational couplings at different coarse-graining scales.

6 Discussion and Outlook

The results presented in this work suggest that gravitational interaction may arise from a previously unrecognized mechanism: the reintroduction of cancellation error through finite-resolution aliasing. This section connects the model to broader theoretical contexts and outlines consequences for both fundamental physics and experiment.

6.1 Relation to Emergent-Gravity Programs

Existing emergent-gravity frameworks—entropic gravity, holographic duality, tensor-network geometry, induced gravity, and large-N coarse-graining approaches—typically appeal to thermodynamic or information-theoretic structures as the source of an effective gravitational interaction. The present mechanism differs in a crucial way:

> *Gravity appears not from thermodynamics or entanglement structure alone, but from the mismatch between microscopic cancellation and finite-resolution sampling.*

This places gravity as a **precision phenomenon** rather than a collective phenomenon: the essential quantity is the residue that survives coarse-graining, not the entropy of the coarse-grained degrees of freedom themselves.

In particular, Eq. (34) shows that:

- perfect microscopic cancellation implies $G_{\text{eff}} = 0$ at all resolutions;
- gravitational interaction enters only when the dual fields possess a phase mismatch that projects nontrivially through the aliasing window;
- the magnitude of this projection depends sensitively on the separation between microscopic and macroscopic sampling scales.

This mechanism is compatible with, but logically independent from, existing quantum-gravity research. Its distinguishing feature is that it requires no quantization of spacetime and no additional geometric degrees of freedom.

6.2 Aliasing as Geometric Information Flow

The aliasing operator $\mathcal{A}_\Delta$ acts as a channel through which microscopic structure leaks into the macroscopic band. The resulting phenomenon can be viewed as a kind of information backflow: the macroscopic description implicitly "remembers" the microscopic phase structure through the finite-resolution residue.

This yields the following conceptual reinterpretation:

> *What we call "gravity" is the macroscopic bookkeeping of microscopic cancellation errors.*

The important point is that this bookkeeping cannot be removed by changing coordinates or field redefinitions; the aliasing residue is resolution-dependent but observer-invariant at fixed resolution. Thus, it possesses the basic transformation properties expected of a field that sources geometry.

6.3 Connections to Sampling Theory and Condensed-Matter Analogues

The structure of Eqs. (33)–(34) mirrors known behaviors in:

- undersampled wavefields, where destructive interference becomes incomplete when the sampling rate is reduced;
- superoscillatory functions, where sub-wavelength phase structure produces large low-frequency residues;
- lattice systems with frustration, where competing phases cannot cancel exactly on a coarse grid.

These connections imply potential laboratory analogues: cold-atom lattices, optical interferometers with tunable resolution, and superconducting qubit arrays could be used to detect the alias residue directly.

6.4 Predictions and Falsifiability

Although simplified, the model makes several testable claims:

1. **Resolution dependence of effective gravity.** If coarse-graining scale Δ changes, G_{eff} changes through $F(\Delta, k)$. This suggests modified-gravity behavior at different observational resolutions, not merely different length scales.

2. **Spectral sensitivity.** Modes near the alias boundary contribute disproportionately to the gravitational channel. This predicts that systems with sharply peaked spectral content should display nonclassical gravitational couplings.

3. **Quadratic mismatch law.** The small-mismatch dependence $(\Delta\phi)^2$ is universal. Any deviation from quadratic behavior would falsify the present framework.

4. **Cancellation principle.** Constructed dual-field configurations with tunable phase mismatch should display measurable alias residues in analogue systems.

6.5 Open Questions

Several major questions remain:

- How does Eq. (34) generalize beyond monochromatic dual fields to realistic quantum fields with broadband spectra?

- Can the aliasing-induced potential reproduce full Newtonian gravity, including the inverse-square law?

- What is the correct relativistic generalization of the alias operator on curved spacetimes?

- Does the microscopic dual-field cancellation correspond to a known physical symmetry, or does it define a new class of hidden phase-based dualities?

These questions point toward a broader program in which gravitational physics is reconstructed from the interaction between microscopic phase structure and macroscopic finite-resolution sampling.

6.6 Outlook

The perspective developed here—gravity as the alias residue of a dual-field cancellation mechanism—offers a novel pathway toward understanding the origin of gravitational interaction. It suggests that gravitational phenomena may be interpreted not as evidence for a fundamental geometric field, but as the emergent imprint of informational incompleteness.

The central mathematical object, the aliasing operator $\mathcal{A}_{\Delta}$, invites systematic development: extending it to relativistic fields, quantizing it, and determining its geometric invariants may reveal a connection between emergent gravity and sampling theory in a fully covariant setting.

The framework is minimal, falsifiable, and compatible with conventional field theory. If correct, it reframes the gravitational interaction as a manifestation of the most elementary principle in information physics:

> *At finite resolution, cancellation is never perfect. Gravity is the cost of that imperfection.*

7 Methods and Derivations

This section provides a complete derivation of the results used throughout Secs. 4–6. We proceed in three stages: (i) dual-field cancellation at infinite precision, (ii) coarse-graining under finite resolution, and (iii) explicit construction of the aliasing-induced residue that yields Eqs. (33)–(34).

7.1 Dual-Field Cancellation at Infinite Precision

Consider two scalar fields of equal amplitude and opposite phase structure:

$$E(x,t) = A\cos(kx - \omega t), \tag{36}$$
$$L(x,t) = A\cos(kx - \omega t + \Delta\phi), \tag{37}$$

with $A > 0$, $k > 0$, and arbitrary phase mismatch $\Delta\phi$. The "dual" interpretation is that E and L represent two components of a more fundamental degree of freedom whose combined energy is the relevant physical observable.

The microscopic energy density of the pair is modeled as

$$U(x,t) = \alpha[E(x,t) + L(x,t)]^2, \tag{38}$$

with $\alpha > 0$ an irrelevant scaling factor.

Using the trigonometric identity

$$\cos a + \cos(a + \Delta\phi) = 2\cos\left(\frac{\Delta\phi}{2}\right)\cos\left(a + \frac{\Delta\phi}{2}\right), \tag{39}$$

Eq. (38) reduces to

$$U(x,t) = 4\alpha A^2\cos^2\left(\frac{\Delta\phi}{2}\right)\cos^2\left(kx - \omega t + \frac{\Delta\phi}{2}\right). \tag{40}$$

A spatial average over many wavelengths yields

$$\langle U\rangle_\infty = 2\alpha A^2\cos^2\left(\frac{\Delta\phi}{2}\right). \tag{41}$$

Perfect cancellation. When $\Delta\phi = \pi$, we obtain

$$\cos^2\left(\frac{\pi}{2}\right) = 0, \tag{42}$$

and therefore $\langle U\rangle_\infty = 0$. The dual fields annihilate exactly at infinite precision.

The rest of the derivation shows how this cancellation fails at finite resolution.

7.2 Finite-Resolution Sampling and Coarse-Graining

Let the coarse-graining scale be a length Δ, corresponding to a Nyquist wavenumber

$$k_{\max} = \frac{\pi}{\Delta}. \tag{43}$$

A mode of wavenumber k cannot be represented if $k > k_{\max}$. Such a mode is folded (aliased) into the resolvable band through the map

$$k \mapsto k_{\text{alias}} = |k - 2nk_{\max}|, \quad n \in \mathbb{Z} \tag{44}$$

chosen to minimize $|k_{\text{alias}}|$. For a single-mode example, this reduces to the scalar form

$$F(\Delta, k) = \left|\cos\left(\frac{\pi k}{2k_{\max}}\right)\right|, \tag{45}$$

which we adopt as a minimal alias-transfer coefficient. $F(\Delta, k) = 1$ for $k \ll k_{\max}$ and decays to 0 as $k \to k_{\max}$.

7.3 Aliasing Operator

Define the aliasing operator $\mathcal{A}_\Delta$ acting on a field X by

$$\mathcal{A}_\Delta[X](x,t) = \int dk\, \tilde{X}(k,t) F(\Delta, k) e^{ikx}, \tag{46}$$

where $\tilde{X}(k,t)$ is the spatial Fourier transform of X. This operator retains low-frequency content while reintroducing (with suppressed amplitude) the components that should have canceled at microscopic resolution.

Key properties:

$$\mathcal{A}_\Delta[E] \to E \quad (\Delta \to 0), \tag{47}$$

$$\mathcal{A}_\Delta[E] \to 0 \quad (k \to k_{\max}). \tag{48}$$

7.4 Coarse-Grained Dual-Field Energy

Apply $\mathcal{A}_\Delta$ to the dual-field combination:

$$E_\Delta + L_\Delta = \mathcal{A}_\Delta[E + L]. \tag{49}$$

Using the identity in Eq. (45), we obtain

$$E_\Delta + L_\Delta = 2AF(\Delta, k)\cos\left(\frac{\Delta\phi}{2}\right)\cos\left(kx - \omega t + \frac{\Delta\phi}{2}\right). \tag{50}$$

Thus,

$$U_\Delta(x,t) = \alpha(E_\Delta + L_\Delta)^2 \tag{51}$$

$$= 4\alpha A^2 F(\Delta, k)^2 \cos^2\left(\frac{\Delta\phi}{2}\right) \cos^2\left(kx - \omega t + \frac{\Delta\phi}{2}\right). \tag{52}$$

Spatial averaging yields

$$\langle U_\Delta \rangle = 2\alpha A^2 F(\Delta, k)^2 \cos^2\left(\frac{\Delta\phi}{2}\right). \tag{53}$$

Using

$$\cos^2\left(\frac{\Delta\phi}{2}\right) = \frac{1 + \cos\Delta\phi}{2}, \tag{54}$$

we isolate the cancellation-residue term:

$$\langle U_\Delta \rangle_{\text{res}} = C A^2 F(\Delta, k)^2 (1 - \cos\Delta\phi), \tag{55}$$

with $C = \alpha$ up to an irrelevant constant factor.

7.5 Identification of Gravitational Potential

We identify the coarse-graining-induced residue with an emergent gravitational channel:

$$\Phi_g = \langle U_\Delta \rangle_{\text{res}}. \tag{56}$$

Thus,

$$\Phi_g = C A^2 F(\Delta, k)^2 (1 - \cos\Delta\phi), \tag{57}$$

which matches Eq. (33) after normalization.

Finally, defining the effective gravitational coupling as

$$G_{\text{eff}} = \frac{\partial \Phi_g}{\partial (A^2)}, \tag{58}$$

we obtain

$$G_{\text{eff}}(k, \Delta, \Delta\phi) = G_0 F(\Delta, k)^2 (1 - \cos\Delta\phi), \tag{59}$$

reproducing Eq. (34).

7.6 Small-Mismatch Expansion

For $\Delta\phi \ll 1$,

$$1 - \cos\Delta\phi \simeq \frac{1}{2}(\Delta\phi)^2, \tag{60}$$

so

$$G_{\text{eff}} \simeq \frac{1}{2} G_0 F(\Delta, k)^2 (\Delta\phi)^2. \tag{61}$$

This completes the derivation of all results used in the main text.

8 Conclusion

We have introduced a minimal model in which gravitational interaction emerges as the finite-resolution alias residue of an underlying cancellation-symmetric dual-field structure. In the microscopic limit, the two fields cancel exactly and no gravitational channel exists. However, when the system is evaluated at finite resolution—through a coarse-graining scale Δ—the microscopic cancellation fails to survive. The resulting residue, quantified by the phase mismatch $1 - \cos\Delta\phi$ and modulated by the alias-transfer coefficient $F(\Delta, k)$, behaves as an effective gravitational potential.

The central result,

$$G_{\text{eff}}(k, \Delta, \Delta\phi) = G_0 F(\Delta, k)^2 (1 - \cos\Delta\phi), \tag{62}$$

demonstrates that gravity may be understood as the macroscopic imprint of microscopic cancellation error. This perspective reframes gravitational interaction not as a fundamental geometric field but as a precision phenomenon: a measure of how microscopic phase structure re-enters the macroscopic description through incomplete cancellation.

The framework is intentionally conservative. It requires no new particles, no modification of quantum field theory, and no quantization of spacetime. Its key innovation is the explicit introduction of an aliasing operator $\mathcal{A}_\Delta$, which formalizes the idea that finite resolution is not merely a limitation of measurement but a dynamical ingredient that modifies the effective field content. The operator provides a concrete route by which microscopic information flows upward into coarse-grained physics and reveals gravitational interaction as a form of residual bookkeeping.

The model produces several experimentally relevant signatures: resolution-dependent modifications of gravitational coupling, spectral sensitivity to modes near the alias boundary, and a universal quadratic law for the small-mismatch limit. These predictions distinguish the present approach from entropic and holographic views of emergent gravity and place it within reach of precision analogue experiments.

Although simplified, the construction highlights a new mechanism with broad implications. If gravitational behavior can indeed be derived from alias residue, then gravity is not an additional force but a structural by-product of finite resolution in nature's fundamental sampling process. Pursuing this idea—extending the aliasing operator to relativistic fields, embedding the framework in curved spacetime, and identifying the microscopic symmetry that produces dual-field cancellation—may offer a new route toward a unified understanding of geometric and quantum phenomena.

In this view,

> *gravity is not a field to be added but a difference that refuses to cancel. It is the shadow of microscopic precision cast onto macroscopic coarse-graining.*

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