Quantum Aliasing Field Theory: A Gauge-Invariant Action Framework for Observer-Dependent Dynamics

Benjamin N. Miller
Independent Researcher
itsbenagain@gmail.com

December 2025

Abstract

We introduce a field-theoretic framework in which measurement, phase selection, and state-reduction phenomena emerge from a gauge-invariant aliasing field coupled to standard quantum degrees of freedom. The theory begins from the premise that the observer's state imposes a dynamically relevant constraint on phase resolution, modeled as an effective nonlocal field with its own action, symmetries, and stress–energy contribution. We construct a Lorentz-invariant action containing: (i) the standard matter-field Lagrangian, (ii) a dynamical aliasing field that encodes resolution-limited phase information, and (iii) a coupling term that modifies the evolution of the wavefunction without violating gauge invariance. The resulting Euler–Lagrange equations yield a modified Schrödinger–Dirac evolution rule whose deviations from linearity depend on the geometry of the aliasing field and its interaction with measurement settings. This formalism recovers ordinary quantum mechanics as a limiting case but predicts distinct correlation structures under conditions of constrained phase bandwidth. We outline the theoretical structure, symmetries, and observational consequences, establishing a mathematical foundation for "quantum aliasing" as a physically testable extension of standard quantum theory.

1 Introduction

Quantum theory provides an extraordinarily successful operational framework, yet its formal structure contains well-known ambiguities regarding state reduction, contextuality, and the status of phase information under measurement. Standard formulations treat the wavefunction as a complete descriptor of physical systems, while the measurement process is modeled through external rules that do not arise from the unitary dynamics itself. Attempts to modify or extend quantum mechanics often introduce nonlinearities, stochastic collapse mechanisms, or hidden-variable structures, but typically at the expense of gauge symmetry, Lorentz covariance, or empirical consistency.

This work develops an alternative approach based on the hypothesis that measurement-induced discontinuities can be understood as aliasing effects arising from finite phase resolution encoded in an additional dynamical field. Instead of regarding interference breakdown or state selection as an extrinsic or ad hoc process, we treat aliasing as an intrinsic geometric feature of the underlying action. The formalism introduces a gauge-invariant field $\mathcal{A}_\mu(x)$ whose role is to parameterize bandwidth constraints on phase information and whose coupling modifies the evolution of matter fields while preserving locality at the level of the action. The resulting theory retains linearity in the appropriate limit, reduces to standard quantum mechanics when aliasing effects vanish, and remains compatible with relativistic covariance.

We construct the full action, derive the coupled Euler–Lagrange equations, identify the conserved currents and symmetries, and examine how the theory departs from standard predictions. Although the present work focuses on the theoretical foundation, the framework generates experimentally accessible deviations in phase-sensitive correlation measurements, particularly where resolution constraints play a dominant role.

This paper is organized as follows:

- Section 2 reviews the background and theoretical context.
- Section 3 defines the aliasing field, its symmetries, and its gauge structure.
- Section 4 presents the total action and derives the modified evolution equations.
- Section 5 analyzes observational signatures and limiting cases.
- Section 6 provides a broader discussion and interpretation.
- Section 7 outlines future directions and open questions.
- The Appendix collects mathematical details and an example system.

2 Background and Theoretical Context

Quantum theory is both empirically complete and conceptually incomplete. Its mathematical structure predicts interference, entanglement, and correlation with extraordinary precision, yet it remains silent on the mechanisms by which phase information is lost or transformed during measurement. The standard formalism separates the theory into two incompatible components:

1. unitary evolution governed by linear differential equations, and
2. state reduction governed by rules imposed externally.

This dual structure is operationally effective but conceptually unstable: the collapse postulate is not derived from the dynamics, does not appear in the action, and does not arise from any symmetry or conservation principle.

Several major frameworks have attempted to resolve this tension:

- Hidden-variable theories introduce unobservable degrees of freedom but often conflict with locality or gauge invariance.
- Spontaneous-collapse models modify dynamics through nonlinear or stochastic terms, but typically violate relativistic covariance or alter energy conservation.
- Decoherence-based approaches correctly describe environmental suppression of interference, yet they do not explain how a unique outcome is selected.
- Relational and observer-dependent models retain operational consistency but lack a corresponding field-theoretic formulation.

Across these approaches, one structural feature remains conspicuously absent: a dynamical description of the loss of phase resolution itself. Interference breakdown is always modeled as a consequence of hidden variables, environmental entanglement, nonlinear collapse, or information transfer—but never as a geometric constraint on what phases can be resolved.

This absence is notable because every physical measurement device, from optical detectors to biological perceptual systems, possesses a finite bandwidth for resolving phase. The mathematical formalism of quantum mechanics, however, implicitly assumes infinite phase resolution unless collapse is introduced by hand.

The central proposal of quantum aliasing field theory is that phase-resolution constraints are not epistemic or instrumental but dynamical. We model this by introducing a new geometric field $\mathcal{A}_\mu(x)$ that encodes observer-dependent limits on resolvable phase information. Unlike internal gauge fields, $\mathcal{A}_\mu$ does not represent an interaction associated with a gauge group; rather, it captures the coarse-graining structure imposed by any physical measurement context.

This framing places quantum aliasing within a rigorous theoretical lineage:

- Like standard gauge theory, it introduces a covariant structure with curvature.
- Like classical measurement theory, it acknowledges finite bandwidth.
- Like field-theoretic extensions of quantum mechanics, it remains local at the level of the action.
- Like collapse theories, it produces state selection—but without violating gauge symmetry or Lorentz invariance.

Quantum mechanics is recovered as the limit in which $\mathcal{F}_{\mu\nu} = 0$ and phase resolution is uniform. Deviations occur only when the aliasing curvature becomes large, either spontaneously or through interaction with matter fields.

This contextualizes the aliasing framework in relation to decades of theoretical work while identifying its novelty: the dynamical treatment of phase-resolution bandwidth as a geometric field.

3 The Aliasing Field: Definition, Symmetries, and Gauge Structure

The distinguishing feature of the present framework is the introduction of a new dynamical degree of freedom—the **aliasing field**—which captures the finite phase-resolution structure inherent to any observer–system interaction. This field, denoted by $\mathcal{A}_\mu(x)$, does not represent an internal gauge connection associated with a group acting on matter fields, but instead encodes geometric constraints on the resolvable phase information of the wavefunction:

$$\mathcal{A}_\mu(x) : \mathbb{R}^{1,3} \to \mathbb{R}. \tag{1}$$

We postulate that $\mathcal{A}_\mu$ transforms under the $U(1)$-like shift symmetry

$$\mathcal{A}_\mu(x) \to \mathcal{A}_\mu(x) + \partial_\mu \Lambda(x), \tag{2}$$

for any smooth scalar function $\Lambda(x)$. This ensures that only curvature-like combinations of $\mathcal{A}_\mu$ are physically meaningful, prevents the introduction of preferred frames, and maintains compatibility with Lorentz invariance.

The corresponding field strength is defined as

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu, \tag{3}$$

which is gauge invariant. Regions with $\mathcal{F}_{\mu\nu} = 0$ correspond to uniform phase resolution, in which case aliasing effects vanish and the dynamics reduce to standard quantum mechanics. Nonzero curvature encodes gradients in phase bandwidth and constitutes the source of deviations from linear evolution.

The aliasing field couples to matter through the modified covariant derivative

$$\tilde{D}_\mu \psi = \left(\partial_\mu + iA_\mu + i\alpha \mathcal{A}_\mu\right) \psi, \tag{4}$$

where A_μ is the ordinary electromagnetic gauge potential of the matter sector and α is a dimensionless coupling constant controlling the strength of aliasing effects. This preserves the internal gauge symmetry of the matter field as well as the aliasing-gauge symmetry.

Notation clarification: Throughout this paper, A_μ denotes the standard electromagnetic (or other internal) gauge field, while $\mathcal{A}_\mu$ denotes the new aliasing field introduced in this work. These are distinct dynamical fields with different physical interpretations.

Operationally:

- If $\mathcal{A}_\mu$ is small or slowly varying, the dynamics reduce to standard unitary evolution.
- If $\mathcal{A}_\mu$ becomes large or develops sharp gradients (e.g. during measurement), the effective phase bandwidth collapses, suppressing interference and enforcing deterministic, symmetry-consistent state selection.

Unlike hidden-variable proposals, the aliasing field is a genuine part of the action with its own stress–energy tensor and field equations. Measurement is not appended to the theory; it is a geometric phenomenon arising from the structure and dynamics of $\mathcal{A}_\mu$.

4 Total Action and Modified Evolution Equations

We now place the aliasing field on fully dynamical footing by constructing a Lorentz-invariant action with three sectors: the standard matter Lagrangian, the aliasing-field Lagrangian, and a gauge-invariant coupling between them. The total action is

$$S = \int d^4x \left(\mathcal{L}_{\text{matter}} + \mathcal{L}_{\mathcal{A}} + \mathcal{L}_{\text{coupling}} \right). \tag{5}$$

4.1 Matter Sector

For definiteness we treat the matter field as a Dirac spinor, although the structure generalizes. The standard Dirac Lagrangian is

$$\mathcal{L}_{\text{matter}} = \bar{\psi} \left(i\gamma^\mu D_\mu - m \right) \psi, \tag{6}$$

with

$$D_\mu = \partial_\mu + iA_\mu, \tag{7}$$

the conventional gauge-covariant derivative, where A_μ is the electromagnetic gauge field.

4.2 Aliasing-Field Sector

The aliasing field is a real four-vector with shift symmetry

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu \Lambda, \tag{8}$$

which guarantees that only curvature-like combinations are physical. The simplest gauge-invariant Lagrangian is

$$\mathcal{L}_{\mathcal{A}} = -\frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - V\left(\mathcal{A}_\mu \mathcal{A}^\mu \right), \tag{9}$$

where

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu, \tag{10}$$

and V is an optional nonlinear self-interaction potential enabling richer measurement-like dynamics.

4.3 Coupling Sector

Aliasing acts by modifying phase resolution, suggesting that matter fields should couple only through their phase. This is implemented by replacing D_μ with

$$\tilde{D}_\mu \psi = \left(\partial_\mu + iA_\mu + i\alpha \mathcal{A}_\mu \right) \psi, \tag{11}$$

leading to the interaction term

$$\mathcal{L}_{\text{coupling}} = \alpha \bar{\psi} \gamma^\mu \mathcal{A}_\mu \psi. \tag{12}$$

This preserves all gauge symmetries and introduces no preferred frames.

4.4 Euler–Lagrange Equations

Variation with respect to $\bar{\psi}$ gives the aliasing-modified Dirac equation:

$$\left(i\gamma^{\mu}\tilde{D}_{\mu} - m\right)\psi = 0, \tag{13}$$

or explicitly,

$$\left(i\gamma^{\mu}D_{\mu} + \alpha\gamma^{\mu}\mathcal{A}_{\mu} - m\right)\psi = 0. \tag{14}$$

Variation with respect to $\mathcal{A}_{\mu}$ yields the field equation:

$$\partial_{\nu}\mathcal{F}^{\nu\mu} + \frac{\partial V}{\partial \mathcal{A}_{\mu}} = \alpha j^{\mu}, \tag{15}$$

where

$$j^{\mu} = \bar{\psi}\gamma^{\mu}\psi \tag{16}$$

is the conserved matter current.

4.5 Interpretation

The coupled system exhibits two regimes:

- If $\mathcal{A}_{\mu}$ is uniform or small, the coupling term is negligible and standard quantum dynamics are recovered.
- If $\mathcal{A}_{\mu}$ develops strong gradients or large amplitude—as during measurement—the aliasing term suppresses interference by reducing locally available phase bandwidth, inducing deterministic, gauge-consistent state selection.

Thus measurement becomes an ordinary physical interaction arising from the geometry and dynamics of $\mathcal{A}_{\mu}$, not an external rule appended to quantum mechanics.

5 Observational Signatures

The aliasing field $\mathcal{A}_{\mu}$ introduces experimentally accessible deviations from standard quantum mechanics whenever the local phase bandwidth departs from uniformity. These effects arise not from environmental noise or hidden variables, but from curvature in the aliasing field,

$$\mathcal{F}_{\mu\nu} = \partial_{\mu}\mathcal{A}_{\nu} - \partial_{\nu}\mathcal{A}_{\mu}, \tag{17}$$

which determines how finely relative phase can be resolved. The following domains provide the clearest opportunities to test the theory.

5.1 Phase-Sensitive Interferometry

When $\mathcal{F}_{\mu\nu}$ varies across the coherence length of an interferometric system, the modified evolution equation induces coherent, geometry-driven distortions of interference patterns. Predicted signatures include:

- shifts in accumulated phase along interferometric paths,
- saturation or plateauing of fringe visibility at large aliasing amplitude,
- deviations from linear scaling of visibility with path length or enclosed area,
- attenuation of long-baseline coherence without decoherence.

Unlike stochastic decoherence, these effects preserve purity and arise solely from bandwidth limits.

5.2 Entanglement and Correlation Structure

Because the aliasing field couples through the matter current $j^\mu = \bar{\psi}\gamma^\mu\psi$, multipartite states experience global bandwidth constraints. As a result, the theory predicts:

- distortions in Bell-type correlation envelopes under rapid setting changes,
- deviations from sinusoidal polarization or spin-correlation dependence,
- suppression of higher-order interference without loss of entanglement,
- slow, curvature-driven non-Markovian drift in correlations as $\mathcal{A}_\mu$ evolves.

No-signalling is preserved; only correlation strengths and functional forms are modified.

5.3 Deterministic Collapse from Bandwidth Gradients

A distinctive feature of quantum aliasing is deterministic, curvature-induced collapse in isolated systems. Collapse corresponds not to environmental noise but to the onset of sharp gradients in $\mathcal{A}_\mu$:

$$\partial_\mu \mathcal{A}_\nu \text{ large.} \tag{18}$$

This leads to:

- measurement-like state selection without stochasticity,
- predictable breakdown of interference at curvature thresholds,
- hysteresis-like behavior when curvature persists after measurement.

Superconducting qubits, trapped ions, and massive-particle interferometers offer especially clean tests.

5.4 Nonlinear Phase-Bandwidth Threshold Effects

Because aliasing interacts nonlinearly with the local matter current, the theory predicts threshold phenomena absent in linear quantum mechanics:

- photon-flux or intensity thresholds triggering interference breakdown,
- nonlinear suppression in multi-photon interference experiments,
- saturation in high-precision phase encoding at large aliasing amplitude.

These differ from decoherence, which produces smooth damping rather than threshold structure.

5.5 Propagation in Regions of Varying Curvature

Spatial variation in $\mathcal{A}_\mu$ induces bandwidth-driven modifications to propagation:

- group-velocity shifts correlated with curvature gradients,
- curvature-dependent diffraction patterns,
- attenuation of long-range coherence without energy loss.

5.6 Flat-Aliasing Limit

A stringent internal consistency check arises in the limit

$$\mathcal{F}_{\mu\nu} = 0. \tag{19}$$

Here the aliasing field is pure gauge, the generalized covariant derivative reduces to the standard one, and all deviations from quantum mechanics vanish exactly. Controlled flat-curvature regimes therefore provide a clean route to experimental falsification of the theory.

6 Discussion and Interpretation

The aliasing framework developed in this work provides a concrete dynamical mechanism through which phase-resolution constraints influence quantum evolution. Unlike stochastic-collapse models or hidden-variable theories, the aliasing field does not introduce fundamental noise, superluminal signaling, or preferred frames. Instead, it operates through a gauge-invariant geometric constraint that modifies the effective phase bandwidth available to physical systems during evolution and measurement.

At the conceptual level, the theory reframes quantum state reduction as a bandwidth-limiting transition governed by the dynamics of $\mathcal{A}_\mu$. Collapse corresponds not to a nonunitary axiom but to the onset of aliasing curvature when the system's relative-phase structure exceeds the resolvable limit encoded in the field. This allows measurement to be treated as an ordinary interaction within a unified action framework.

The framework offers several advantages over existing approaches to modifying or extending quantum theory:

- **Gauge Symmetry Preserved:** The aliasing field is constructed to maintain internal $U(1)$ symmetry and Lorentz invariance at the level of the action.
- **Deterministic Dynamics:** Deviations from linearity emerge solely from the geometry of $\mathcal{A}_\mu$, avoiding stochasticity or hidden variables.
- **Continuity With Standard Theory:** When $\mathcal{F}_{\mu\nu} = 0$, the formalism recovers conventional quantum mechanics exactly.
- **Action-Based Unification:** Measurement, interference breakdown, and phase suppression are described by a single Lagrangian rather than multiple postulates.

The theory also raises natural questions for further study. The self-interaction potential $V(\mathcal{A}^2)$ plays a key role in determining whether aliasing curvature exhibits threshold behavior, metastability, or hysteresis. Its form governs whether collapse-like transitions are sharp or continuous, and whether aliasing curvature propagates, dissipates, or becomes localized. Identifying physically motivated potentials—possibly inspired by information-theoretic or thermodynamic considerations—remains an open challenge.

Another area of interest is the behavior of multipartite systems. Because the aliasing field couples to the matter current j^μ, its dynamics may involve global constraints in entangled states. Understanding the interplay between aliasing curvature and nonlocal correlation structure could clarify how the theory maintains no-signalling while still modifying interference patterns.

Finally, the theory suggests a conceptual reinterpretation of phase information in quantum mechanics. If phase resolution is fundamentally limited by a dynamical geometric field, then quantum mechanics may be understood as the flat-curvature limit of a more general bandwidth-governed theory. This view opens the door to new connections between quantum foundations, information theory, and geometric field dynamics.

Overall, quantum aliasing offers a coherent and mathematically consistent pathway for embedding measurement, collapse, and phase-resolution phenomena within a single gauge-invariant field-theoretic framework. The next steps involve identifying concrete potentials, exploring multi-system dynamics, and designing experiments sensitive to aliasing curvature—tasks which together will determine whether the theory represents a modest extension of quantum mechanics or a deeper structural generalization.

7 Future Directions and Open Questions

The framework developed above establishes the aliasing field as a new geometric degree of freedom capable of modifying quantum evolution without breaking gauge invariance or relativistic covariance. Several natural extensions follow from this construction, each offering opportunities to refine the theory and connect it more directly to experiment.

A first direction involves the systematic classification of admissible self-interaction potentials $V(\mathcal{A}_\mu \mathcal{A}^\mu)$ in the aliasing-field sector. Nonlinear terms may support richer collapse-like dynamics, including metastable curvature domains or phase-resolution attractors. Understanding their stability and renormalization behavior is essential for delineating the range of viable aliasing models.

A second direction concerns the coupling between $\mathcal{A}_\mu$ and many-body systems. Because the aliasing current $j^\mu_{\text{alias}} = \bar{\psi}\gamma^\mu\psi$ depends on the global structure of the quantum state, large entangled systems may generate collective aliasing curvature. This could lead to emergent bandwidth constraints not visible in single-particle experiments. Developing effective descriptions of this regime is an important next step.

Another open question concerns the interplay between aliasing dynamics and gravity. Since $\mathcal{A}_\mu$ carries stress–energy, its curvature contributes to the Einstein equations. Whether such contributions play a role in semiclassical gravity or in the backreaction associated with measurement processes remains to be explored.

Finally, fully characterizing the experimental signatures requires numerical simulation of the nonlinear feedback between $\mathcal{A}_\mu$ and matter currents. Interferometric platforms, superconducting circuits, and optomechanical systems provide promising testbeds, but quantitative predictions will depend on the detailed dynamics of curvature formation in realistic measurement scenarios.

Overall, the aliasing-field framework opens a novel space of gauge-consistent modifications to quantum theory, offering a unified description of interference breakdown, phase-resolution limits, and measurement-induced dynamics. The results presented here provide a foundation for further theoretical development and experimental investigation.

8 Conclusion

The aliasing-field framework developed in this work establishes a gauge-invariant and Lorentz-covariant extension of quantum theory in which phase-resolution limits arise from a dynamical geometric field rather than ad hoc measurement postulates. Within this formalism, breakdown of interference and collapse-like behavior emerge when the aliasing curvature $\mathcal{F}_{\mu\nu}$ becomes nonuniform over the coherence structure of a system, providing a deterministic and internally consistent mechanism for state selection.

A key outcome of the construction is that standard quantum mechanics is recovered exactly in the flat-aliasing limit $\mathcal{F}_{\mu\nu} = 0$, ensuring continuity with existing experimental results. When curvature develops, the theory predicts specific modifications to phase correlations, entanglement structure, and measurement-induced transitions, all while preserving no-signalling and avoiding stochasticity or hidden variables.

The aliasing field's self-interaction potential $V(\mathcal{A}^2)$ plays a central role in governing the sharpness, stability, and propagation of curvature domains. Its form determines whether bandwidth collapse occurs smoothly or through threshold dynamics, and whether aliasing curvature may persist, dissipate, or localize during measurement interactions. Further study of these potentials is essential to fully characterize the framework.

In multipartite systems, the coupling of the aliasing field to the matter current suggests that collective curvature effects may arise, especially in highly entangled states. Understanding how such global bandwidth constraints interact with correlation structure remains a central open problem, with implications for both foundational physics and large-scale quantum technologies.

Taken together, these results present a coherent field-theoretic account of phase-resolution limits, interference breakdown, and collapse-like phenomena. The theory offers a unified action-based description of measurement and quantum evolution, opening a space of experimentally testable predictions and guiding future exploration of the geometric and informational structure underlying quantum mechanics.

A Mathematical Details and Example System

A.1 Euler–Lagrange Equations

For convenience we collect the relevant pieces of the action discussed in the main text. In units $\hbar = c = 1$ the total action is

$$S = \int d^4x \left(\mathcal{L}_{\text{matter}} + \mathcal{L}_{\mathcal{A}} + \mathcal{L}_{\text{coupling}}\right), \tag{20}$$

with

$$\mathcal{L}_{\text{matter}} = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi, \tag{21}$$

$$\mathcal{L}_{\mathcal{A}} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - V\left(\mathcal{A}_\mu\mathcal{A}^\mu\right), \tag{22}$$

$$\mathcal{L}_{\text{coupling}} = \bar{\psi}\gamma^\mu(i\alpha\mathcal{A}_\mu)\psi, \tag{23}$$

where

$$D_\mu = \partial_\mu + iA_\mu, \quad \mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu. \tag{24}$$

The total covariant derivative acting on the matter field can be written as

$$\tilde{D}_\mu\psi = \left(\partial_\mu + iA_\mu + i\alpha\mathcal{A}_\mu\right)\psi. \tag{25}$$

A.1.1 Variation with respect to $\bar{\psi}$

The Euler–Lagrange equation for $\bar{\psi}$ is

$$\frac{\partial\mathcal{L}}{\partial\bar{\psi}} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\right) = 0. \tag{26}$$

Using the expressions above we obtain

$$\left(i\gamma^\mu D_\mu + \alpha\gamma^\mu\mathcal{A}_\mu - m\right)\psi = 0. \tag{27}$$

Equivalently,

$$\left(i\gamma^\mu\tilde{D}_\mu - m\right)\psi = 0, \tag{28}$$

which is the aliasing-modified Dirac equation used in the main text.

A.1.2 Variation with respect to $\mathcal{A}_\mu$

The Euler–Lagrange equation for $\mathcal{A}_\mu$ reads

$$\frac{\partial \mathcal{L}}{\partial \mathcal{A}_\mu} - \partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \mathcal{A}_\mu)} \right) = 0. \tag{29}$$

From $\mathcal{L}_{\mathcal{A}}$ we have

$$\frac{\partial \mathcal{L}_{\mathcal{A}}}{\partial(\partial_\nu \mathcal{A}_\mu)} = -\frac{1}{2}\mathcal{F}^{\nu\mu}, \tag{30}$$

$$\frac{\partial \mathcal{L}_{\mathcal{A}}}{\partial \mathcal{A}_\mu} = -\frac{\partial V}{\partial \mathcal{A}_\mu}. \tag{31}$$

From $\mathcal{L}_{\text{coupling}}$ we obtain

$$\frac{\partial \mathcal{L}_{\text{coupling}}}{\partial \mathcal{A}_\mu} = \alpha \bar{\psi}\gamma^\mu \psi \equiv \alpha j^\mu_{\text{alias}}. \tag{32}$$

The field equation for $\mathcal{A}_\mu$ is therefore

$$\partial_\nu \mathcal{F}^{\nu\mu} + \frac{\partial V}{\partial \mathcal{A}_\mu} = \alpha j^\mu_{\text{alias}}, \tag{33}$$

where $j^\mu_{\text{alias}} = \bar{\psi}\gamma^\mu\psi$ is the aliasing current.

A.2 Conserved Currents and Flat-Curvature Limit

The matter sector is invariant under the global $U(1)$ phase transformation

$$\psi \to e^{i\theta}\psi, \quad \bar{\psi} \to \bar{\psi}e^{-i\theta}, \tag{34}$$

with A_μ and $\mathcal{A}_\mu$ unchanged. Noether's theorem yields the conserved current

$$j^\mu = \bar{\psi}\gamma^\mu\psi, \tag{35}$$

satisfying

$$\partial_\mu j^\mu = 0 \tag{36}$$

whenever the equations of motion hold. The aliasing coupling does not break this global symmetry; it only modifies the local evolution via $\mathcal{A}_\mu$.

The aliasing field itself enjoys the gauge symmetry

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \partial_\mu \Lambda(x), \tag{37}$$

under which $\mathcal{F}_{\mu\nu}$ is invariant. In regions where

$$\mathcal{F}_{\mu\nu} = 0, \tag{38}$$

the field is locally pure gauge and can be written as $\mathcal{A}_\mu = \partial_\mu \Lambda$. In this flat-curvature limit the aliasing contribution can be absorbed into a redefinition of the phase of ψ, and the dynamics reduce exactly to standard quantum mechanics.

A.3 Example System: Free Dirac Particle in a Static Aliasing Curvature Bump

To illustrate the effect of the aliasing field on a concrete system, consider a single Dirac particle in (1 + 1) dimensions with no ordinary gauge potential ($A_\mu = 0$) and a static aliasing profile. The Lagrangian density reduces to

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + \bar{\psi}\gamma^\mu (i\alpha \mathcal{A}_\mu)\psi - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - V(\mathcal{A}_\mu \mathcal{A}^\mu). \tag{39}$$

We choose a simple background configuration

$$\mathcal{A}_0(x) = \mathcal{A}_0(x), \quad \mathcal{A}_1(x) = 0, \tag{40}$$

with $\mathcal{A}_0(x)$ localized in space and static in time. The only non-vanishing component of the curvature is

$$\mathcal{F}_{01}(x) = -\partial_x \mathcal{A}_0(x), \tag{41}$$

so the aliasing curvature is concentrated in the region where $\mathcal{A}_0$ varies.

The modified Dirac equation reads

$$\left(i\gamma^0 \partial_t + i\gamma^1 \partial_x + \alpha\gamma^0 \mathcal{A}_0(x) - m\right)\psi(t,x) = 0. \tag{42}$$

Passing to the Hamiltonian form,

$$i\partial_t \psi(t,x) = H_{\text{eff}}\psi(t,x), \tag{43}$$

with

$$H_{\text{eff}} = -i\gamma^0\gamma^1\partial_x + \gamma^0 m - \alpha\mathcal{A}_0(x). \tag{44}$$

The aliasing field thus appears as an effective scalar potential $V_{\text{alias}}(x) = \alpha\mathcal{A}_0(x)$.

In the non-relativistic limit, writing

$$\psi(t,x) = e^{-imt}\begin{pmatrix}\phi(t,x)\\ \chi(t,x)\end{pmatrix}, \tag{45}$$

and integrating out the small component χ in the usual way, one obtains a Schrödinger-like equation for ϕ,

$$i\partial_t\phi(t,x) = \left[-\frac{1}{2m}\partial_x^2 + V_{\text{alias}}(x)\right]\phi(t,x) + O\left(\frac{1}{m^2}\right). \tag{46}$$

Consider now a wavepacket incident on a region where $\mathcal{A}_0(x)$ forms a localized bump. A superposition of two paths that traverse regions with different integrated aliasing potential,

$$\Delta\Phi_{\text{alias}} = \alpha\int_{\text{path 1}} dt\,\mathcal{A}_0(x(t)) - \alpha\int_{\text{path 2}} dt\,\mathcal{A}_0(x(t)), \tag{47}$$

acquires an additional relative phase due to aliasing. When curvature is weak, $\Delta\Phi_{\text{alias}}$ produces a small, coherent shift of the interference pattern. When the effective phase spread exceeds the resolvable bandwidth encoded in $\mathcal{A}_\mu$, the aliasing field reacts dynamically (through its own equation of motion) and suppresses interference, reproducing the qualitative collapse-like behavior discussed in the main text.

In the limit where $\mathcal{A}_0(x) \to 0$ and $\mathcal{F}_{01}(x) \to 0$ everywhere, the effective potential vanishes, H_{eff} reduces to the free Dirac Hamiltonian, and the usual interference pattern is fully restored. This example illustrates how the same formalism can describe both standard quantum evolution and aliasing-induced deviations within a single, gauge-invariant action.

References

[1] E. Schrödinger, "Quantisierung als Eigenwertproblem," *Annalen der Physik* **79**, 361–376 (1926).

[2] P. A. M. Dirac, "The Quantum Theory of the Electron," *Proc. Roy. Soc. A* **117**, 610–624 (1928).

[3] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Princeton University Press (1932).

[4] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill (1965).

[5] C. N. Yang and R. L. Mills, "Conservation of Isotopic Spin and Isotopic Gauge Invariance," *Phys. Rev.* **96**, 191–195 (1954).

[6] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995).

[7] S. Weinberg, *The Quantum Theory of Fields, Vol. 1*, Cambridge University Press (1996).

[8] W. H. Zurek, "Decoherence and the Transition from Quantum to Classical," *Physics Today* **44**, 36–44 (1991).

[9] W. H. Zurek, "Decoherence, einselection, and the quantum origins of the classical," *Rev. Mod. Phys.* **75**, 715–775 (2003).

[10] G. C. Ghirardi, A. Rimini, and T. Weber, "Unified dynamics for microscopic and macroscopic systems," *Phys. Rev. D* **34**, 470–491 (1986).

[11] P. Pearle, "Combining stochastic dynamical state-vector reduction with spontaneous localization," *Phys. Rev. A* **39**, 2277–2289 (1989).

[12] A. Bassi and G. Ghirardi, "Dynamical reduction models," *Phys. Rep.* **379**, 257–426 (2003).

[13] S. Weinberg, "Precision Tests of Quantum Mechanics," *Phys. Rev. Lett.* **62**, 485–488 (1989).

[14] T. W. B. Kibble, "Relativistic Models of Nonlinear Quantum Mechanics," *Commun. Math. Phys.* **64**, 73–82 (1978).

[15] I. Bialynicki-Birula and J. Mycielski, "Nonlinear Wave Mechanics," *Ann. Phys.* **100**, 62–93 (1976).

[16] T. Frankel, *The Geometry of Physics, 3rd ed.*, Cambridge University Press (2011).

[17] S. M. Carroll, *Spacetime and Geometry*, Addison-Wesley (2004).